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Is the nature of itinerant ferromagnetism playing a role in the competition between spin polarization and singlet pair correlations?

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Abstract
We consider the competition between spin singlet pairing and itinerant ferromagnetism whose magnetization is yielded by a relative shift of the bands with opposite spin polarization or by asymmetric spin-dependent bandwidths. Within the framework of the exact solution of an extended version of the reduced BCS model, the structure of the coexisting state is shown to have general features that are not related to the character of the ferromagnetism. The role of different types of ferromagnet is then investigated for the proximity effect in a system made of a bilayer junction with a spin singlet superconductor interfaced with a ferromagnet in the clean limit. We show that the qualitative behaviour of the proximity effect does not depend on the nature of the ferromagnetism. Differences emerge at the borderline with the half-metallic regime. For the spin-dependent bandwidth type of ferromagnetism the pairing amplitude exhibits an oscillating behaviour until the density of the minority spin carrier becomes almost zero. The crossover from an oscillating to an exponentially damped profile occurs away from the half-metallic limit when a spin exchange type ferromagnet is considered.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The problem of the competition between superconductivity and ferromagnetism (spin imbalance) is attracting a lot of interest within different areas of scientific research from solid state [1–5] to quark matter physics [6, 7]. In this context, some of the questions that one is typically facing are related to (i) the nature of coexisting phases and the way to detect them, (ii) the most favourable conditions for getting a coexisting state, (iii) the role played by the mechanisms giving rise to pairing and spin polarization and (iv) the behaviour of the order parameter within natural or artificially made materials near the interface between a ferromagnetic (FM) and a superconducting (SC) subsystem.

Early observations and related studies on the competition between singlet-type superconductivity and spin polarization have led us to conclude that coexistence has to imply inhomogeneity in the spin density and/or pair density distribution. This is the case for the crypto-ferromagnetism [8, 9], where magnetic domains of a suitable size reduce the strength of the pair breaking, or the so-called Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) state [10, 11] whose modulation in phase or amplitude of the SC order parameter allows for superconductivity in high magnetic fields above the Clogston limit. A similar fate happens to the spatial evolution of the pair amplitude within the ferromagnetic side of a junction made by a ferromagnet interfaced with a superconductor [3]. There, the propagation of Cooper pairs is marked by an oscillating

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behaviour modulated by an exponential function. Two scale lengths, associated with the oscillating and the damped spatial profile, are characteristic of the proximity phenomena and are basically set by the strength of disorder in the ferromagnetic side.

Recently, the interest in this topic has been revived by different observations both for natural or artificially made materials. The discovery of new materials, exhibiting metallic ferromagnetism at a higher critical temperature than the SC one [12–15], has brought attention towards the general problem of pairing in the presence of mismatched Fermi surfaces due to electron–electron correlations. Still, novel features in the proximity effect between a superconductor and a ferromagnet have been found for the case of unconventional pairing [16–18] or near the so-called half-metallic regime [19]. In this context, it is the possibility of tailoring superconductor–ferromagnet junctions with desired features that implies the need for specific modelling for the interface and the superconducting as well as the ferromagnetic components.

Based on such motivations, here we consider the competition of itinerant ferromagnetism and conventional singlet-type superconductivity [20, 21] by focusing on two different mechanisms yielding spin polarization in the ferromagnet (see figure 1). The analysis deals with the case of band-split (FM1) and spin-dependent mass (FM2) ferromagnetism, respectively. In the former, the magnetization is generated by a rigid shift of the band for the spin majority carriers with respect to the minority ones, as it occurs in a Stoner-like ferromagnet. The latter is marked by an asymmetric renormalization of the effective mass or of the bandwidth, depending on the spin orientation of the electrons close to the Fermi level, as it may occur in a ferromagnet driven by kinetic energy gain mechanisms [22, 23]. The purpose of this paper is dual: (i) to provide a general view of the ground state structure when both ferromagnetism and superconductivity occur at the Fermi level and (ii) to analyse and compare the proximity behaviour for a junction made of a ferromagnet–superconductor bilayer assuming a conventional singlet spin pairing for the superconductor and the possibility of having both FM1 and FM2 as itinerant ferromagnets.

This paper is organized as follows. In section 2 we introduce a model Hamiltonian describing pairing in the spin singlet channel and ferromagnetism of types FM1 and FM2, for which we provide a sketch of the exact solution with particular emphasis on the structure of the ground state. In section 3, the problem of the proximity between a conventional singlet-type superconductor and a ferromagnet of types FM1 and FM2 is considered for a bilayer junction. By means of the Bogoliubov–de Gennes scheme, we get the self-consistent solution for the magnetic and the superconducting order parameter at any given lattice site and distance from the interface. Section 4 is devoted to the conclusions.

2. Ground state structure for coexisting singlet pairing and itinerant ferromagnetism: exact solution within a reduced BCS model

To study the competition between singlet pairing in the BCS channel (zero total momentum of the Cooper pairs) and ferromagnetism of types FM1 and FM2 near the Fermi level, one can consider the following model Hamiltonian [21]:

$$H = \sum_{j=1}^{\Omega} \sum_{\sigma=+,-} w_\sigma \epsilon_j \, c^\dagger_{j\sigma} c_{j\sigma} - g \sum_{j,j'} c^\dagger_{j+} c^\dagger_{j-} c_{j'-} c_{j'+} - J \sum_{j,j'} \hat{S}_j \cdot \hat{S}_{j'} \, ,$$

where $c^\dagger_{j\sigma}$ ($c_{j\sigma}$) is the creation (annihilation) operator for an electron on level j and ϵ_j are the single-electron energies. The first term in the Hamiltonian H describes the spin-dependent kinetic energy, with w_σ indicating the factor controlling the bandwidth amplitude for different spin polarizations. The second and third terms of H describe the electron–electron interaction via the pairing coupling g and the FM exchange J, respectively. Here the pairing strength is $g = \lambda d$, with d being the mean level spacing between the single-particle energy levels and λ is a dimensionless coupling constant. The form of H turns out to be very useful for analysing the competition between superconductivity and ferromagnetism in the most general frame. Indeed, the dependence on the size of the system is controlled via the amplitude of d, while the strength of the ferromagnetism with spin-dependent bandwidth is tuned by the relative ratio $w_\downarrow/w_\uparrow$, regardless of the microscopic mechanism that generates it. Still, one can work within the canonical ensemble at fixed total number of electrons, because

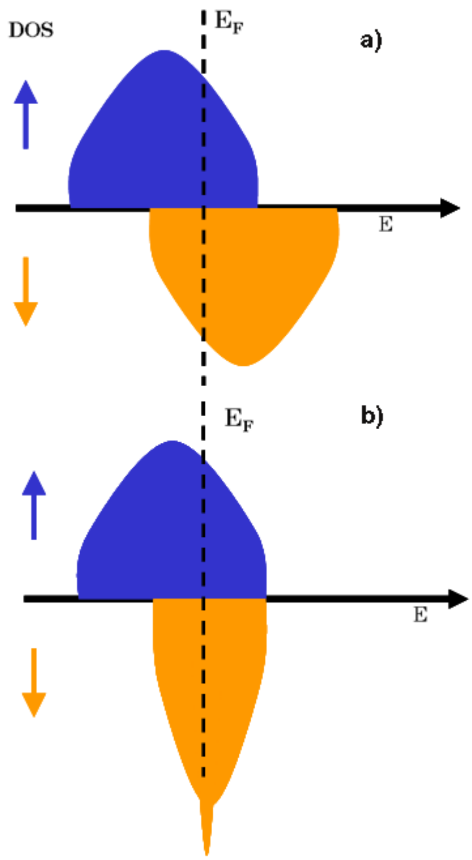

Figure 1. Density of states (DOS) versus energy for (a) the band-split and (b) the spin-dependent bandwidth ferromagnet. E_F stands for the Fermi energy.

the exact solution is built without any explicit breaking of the gauge symmetry as in the usual Hartree decoupling of the interacting term. The control on the size aspect allows us to study the problem from the nanograin case to a macroscopic-like system. Moreover, the possibility of working at a fixed number of electrons can give indications on the role played by the phase fluctuations.

The key point for getting the exact solution of the above model Hamiltonian is based on symmetry arguments. One can show that H can be rearranged in a form where the spin and the pair parts are separated. Indeed, by introducing another $SU(2)$ algebra in the pairing sector $\hat{T}_j^+ = (\hat{T}_j^-)^\dagger = c_{j+}^\dagger c_{j-}^\dagger$, and $\hat{T}_j^z = 1/2(c_{j+}^\dagger c_{j+} + c_{j-}^\dagger c_{j-} - 1)$, one can rewrite the Hamiltonian H into two parts $H = H_T + H_S$, where

$$\begin{aligned} H_T &= \sum_j (w_+ + w_-)\epsilon_j \hat{T}_j^z - \tfrac{1}{2} g \sum_{j,k} (\hat{T}_j^+ \hat{T}_k^- + \hat{T}_k^+ \hat{T}_j^-) \\ H_S &= \sum_j (w_+ w_-)\epsilon_j \hat{S}_j^z - J \sum_{j,j'} \hat{S}_j \cdot \hat{S}_{j'}, \end{aligned} \tag{1}$$

up to an irrelevant constant. Since H_T and H_S commute with each other, the singly occupied levels do not enter into the pair scattering, and thus are blocked for the Pauli principle. Similarly, the double (empty) states do not enter the spin dynamics. The solution is built separately for the spin and the pair part, though the structure for each channel is similar. Let us denote as $|n, m\rangle$ a generic eigenstate of H with $N = 2(n + m)$ electrons. In this state, $2m$ electrons fill a set B of singly occupied (and so blocked) levels, while the remaining n pairs are distributed among the set U of $N_U = \Omega - 2m$ unblocked levels. Hence, following Richardson [25] (see also [26]), one can show that a generic eigenstate of H can be expressed as

$$|n, m\rangle = \prod_{\beta=1}^{m+S^z} |\psi_\beta\rangle \prod_{\mu=1}^{n} |\psi_\mu\rangle$$

where

$$\begin{aligned} |\psi_\beta\rangle &= \sum_{j\epsilon B} \frac{\hat{S}_j^+}{(w_+ - w_-)\epsilon_j - \bar{E}_\beta} |-\rangle \\ |\psi_\mu\rangle &= \sum_{j\epsilon U} \frac{c_{j+}^+ c_{j-}^+}{(w_+ + w_-)\epsilon_j - E_\mu} |0\rangle. \end{aligned}$$

Here $|-\rangle = \prod_{i\epsilon B} c_{i-}^+ |0\rangle$, with $|0\rangle$ being the vacuum state, and S^z is the z projection of the total spin of the electrons in the blocked levels. Furthermore, it is possible to show that the n parameters E_μ and the $m + S^z$ $\bar{E}_\beta$ parameters are the solutions of the two sets of Richardson equations:

$$\begin{aligned} \frac{1}{g} + \sum_{\nu=1(\nu\neq\mu)}^{n} \frac{2}{E_\nu - E_\mu} &= \sum_{j\in U} \frac{1}{(w_+ + w_-)\epsilon_j - E_\mu}, \\ \frac{1}{J} + \sum_{\alpha=1(\alpha\neq\beta)}^{m+S^z} \frac{2}{\bar{E}_\alpha - \bar{E}_\beta} &= \sum_{j\in B} \frac{1}{(w_+ - w_-)\epsilon_j - \bar{E}_\beta}. \end{aligned}$$

In order to have a closer inspection of the structure of the quantum eigenstates, let us start by giving a few simple examples. We will show that there are many features of the quantum configurations that do not depend on the specific

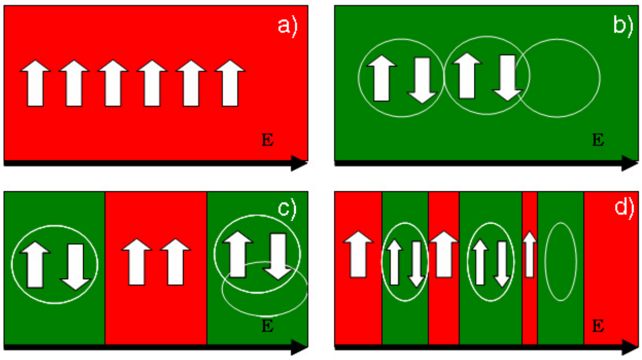

Figure 2. Sketch of the density distribution as a function of the single-particle energy (E) for different quantum eigenstates of H. Panel (a) stands for the FM state with all the spins aligned, panel (b) indicates the pure paired configuration with maximum superconducting correlations, panel (c) shows a SC–FM configuration where there is one blocking sector and panel (d) describes a SC–FM state with many disconnected blocking sectors. The circle contains a pair in a spin singlet configuration. A large (small) arrow in the spin polarized region indicates a configuration with a large (small) magnetization.

mechanism that yields the ferromagnetism. There are two limiting configurations that correspond to the case with zero or maximum magnetization. The pure SC state with $m = 0$ has all the electrons paired and is given by $|n, 0\rangle = \prod_{\mu=1}^{n} |\psi_\mu\rangle$. Otherwise, the configuration FM with all the electrons spin aligned and occupying the lowest energy single-particle levels is expressed as $|0, 2m\rangle = \prod_{\beta=1}^{2m} |\psi_\beta\rangle$. SC is, of course, the ground state when $J = 0$ or $w_\uparrow = w_\downarrow$, while FM is the lowest allowed configuration in the limit of $J \gg (g, d)$ and in the so-called half-metal regime where the bandwidth of the spin minority carrier shrinks to zero, i.e. $w_\downarrow = 0$. It is useful to visualize schematically the density profile in the energy space for the FM and SC states. Their density distribution is either made up of paired (empty) levels or of singly occupied levels. Figures 2(a) and (b) provide a schematic view of the density profile for the states without the coexistence of superconductivity and ferromagnetism.

Let us now consider the structure of the quantum eigenstates exhibiting a coexistence of paired and spin polarized electrons (SC–FM). From a general point of view, the coexistence requires the presence of a blocking sector whose size in energy has to be not equal to the window where the pairing is effective. Such a possibility is schematically depicted in figure 2(c), where the density profile in energy is not uniform and is characterized by a sequence of unblocked–blocked–unblocked sectors. It is worth pointing out that the interface between the blocked and the unblocked sector is sharp in the sense that there is a net separation between the paired and the spin polarized components. The main question about the topology of the SC–FM ground state concerns the distribution of the blocking sectors. Indeed, the character of the SC–FM is completely different depending on whether the blocked sector forms a cluster of singly occupied levels or is made by separated clusters of energy levels that are different in size and disconnected in energy (see figure 2(d), for example). By solving the Richardson equations, one can

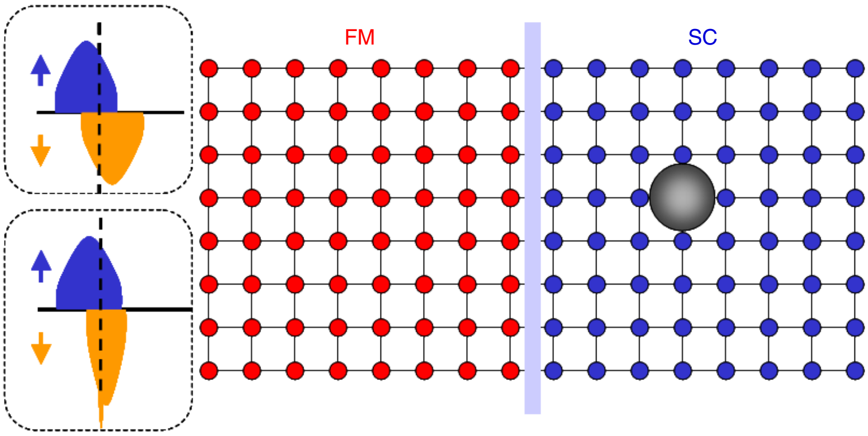

Figure 3. View of the bilayer junction made of a singlet superconductor and a ferromagnet with band-split and spin-dependent bandwidth. The small circles indicate the lattice site of the system while the large circles stand for the s-wave symmetry of the order parameter. The grey rectangle indicates the interface barrier.

show [24] that, if a spin polarization forms within a paired state, the levels that are singly occupied tend to cluster in order to minimize the total energy of the quantum configuration. So far, we did not mention the two different types of ferromagnet. All the arguments we have so far considered are valid independently of the mechanism that generates the spin polarization. Nevertheless, the stability of the SC–FM state with respect to the pure SC and FM ones, as well as the size and the position of the blocked sector, are strongly related to the nature of the itinerant ferromagnet. As a general outcome, it turns out that the ferromagnetism due to a spin-dependent bandwidth allows for a coexistence in a larger region of the space parameters if compared to the case due to a direct magnetic exchange [21].

3. Interfacing singlet s-wave superconductor with different types of itinerant ferromagnet

Let us now consider the question of the competition between superconductivity and ferromagnetism within a proximity system as depicted in figure 3. In this case, we have a two-dimensional bilayer with a ferromagnetic and a superconducting part separated by an interface. In the following, we assume that the direction perpendicular (parallel) to the interface is denoted as x (y) and that the system is uniform along the y-axis direction. The microscopic model is represented by an effective Hubbard model on a square lattice which we treat within the Hartree–Fock approximation to describe magnetism and superconductivity at zero temperature.

Having in mind the previous analysis, our purpose is to explore possible differences emerging in the proximity behaviour comparing the case of ferromagnetism FM1 and FM2. The microscopic Hamiltonian model is made of three components, i.e. the ferromagnetic H_{FM} and the superconducting part H_{SC} plus the interface term H_T. The total Hamiltonian of the system is correspondingly written in the form

$$H = H_{\mathrm{FM}} + H_{\mathrm{SC}} + H_T. \tag{2}$$

The explicit form of the Hamiltonians H_{FM} and H_{SC} is

$$H_A = -\sum_{\langle \mathbf{i},\mathbf{j}\rangle,\sigma} t_{A\sigma}(c^{\dagger}_{\mathbf{i}\sigma}c_{\mathbf{j}\sigma} + \mathrm{h.c.}) + \sum_{\mathbf{i}} U_A n_{\mathbf{i}\uparrow}n_{\mathbf{i}\downarrow} - h_A \sum_{\mathbf{i},\sigma}\left(n_{\mathbf{i}\uparrow} - n_{\mathbf{i}\downarrow}\right) \qquad A = \mathrm{FM}, \mathrm{SC} \tag{3}$$

where $\langle \mathbf{i},\mathbf{j}\rangle$ denotes nearest-neighbour sites, $c_{\mathbf{i}\sigma}$ is the annihilation operator of an electron with spin σ at site $\mathbf{i} \equiv (\mathbf{i_x}, \mathbf{i_y})$ and $n_{\mathbf{i}\sigma} = c^{\dagger}_{\mathbf{i}\sigma}c_{\mathbf{i}\sigma}$ is the corresponding number operator. We include a repulsive local U_{FM} in the FM side to induce a split of the bands as usually occurs in a Stoner ferromagnet, while for the SC part of the junction U_{SC} has a negative amplitude such as to yield s-wave singlet pairing. The magnetic field, which in this context can be equivalently seen either as an external or an intrinsic one, is assumed to be non-vanishing only in the FM side ($h_{\mathrm{FM}} \neq 0$, $h_{\mathrm{SC}} = 0$). Both the interaction U_{FM} and the effective field h_{FM} can be considered as parameters to simulate a ferromagnet with a rigid shift of the majority spin band with respect to the minority one. For the hopping amplitudes we choose $t_{\mathrm{SC}\uparrow} = t_{\mathrm{SC}\downarrow} \equiv t_{\mathrm{SC}}$, with the possibility of having $t_{\mathrm{FM}\uparrow} \neq t_{\mathrm{FM}\downarrow}$. Such a condition is used to reduce the bandwidth for the minority spin electrons with respect to the majority ones so as to reproduce the condition for having an FM2-type ferromagnet.

Finally, the coupling between the two sides of the junction is provided by the term H_T, which is given by

$$H_T = -t_T \sum_{\langle \mathbf{lm}\rangle\sigma}(c^{\dagger}_{\mathbf{l}\sigma}c_{\mathbf{m}\sigma} + \mathrm{h.c.}), \tag{4}$$

where $\mathbf{l}$ ($\mathbf{m}$) denotes sites at the surface for the left (right) layer.

The interaction terms in H_{F} and H_S are decoupled by means of a standard Hartree–Fock approximation such that the magnetic and the pairing channels originate from the on-site and the intersite interactions, respectively:

$$U_{\mathrm{FM}}\, n_{\mathbf{i}\uparrow}n_{\mathbf{i}\downarrow} \simeq U_{\mathrm{FM}}\,[\langle n_{\mathbf{i}\downarrow}\rangle n_{\mathbf{i}\uparrow} + \langle n_{\mathbf{i}\uparrow}\rangle n_{\mathbf{i}\downarrow} - \langle n_{\mathbf{i}\uparrow}\rangle\langle n_{\mathbf{i}\downarrow}\rangle]$$

$$U_{\mathrm{SC}}\, n_{\mathbf{i}\uparrow}n_{\mathbf{i}\downarrow} \simeq U_{\mathrm{SC}}\,[\Delta_{\mathbf{i}}\, c^{\dagger}_{\mathbf{i}\downarrow}c^{\dagger}_{\mathbf{i}\uparrow} + \Delta^{*}_{\mathbf{i}}\, c_{\mathbf{i}\uparrow}c_{\mathbf{i}\downarrow} - |\Delta_{\mathbf{i}}|^2].$$

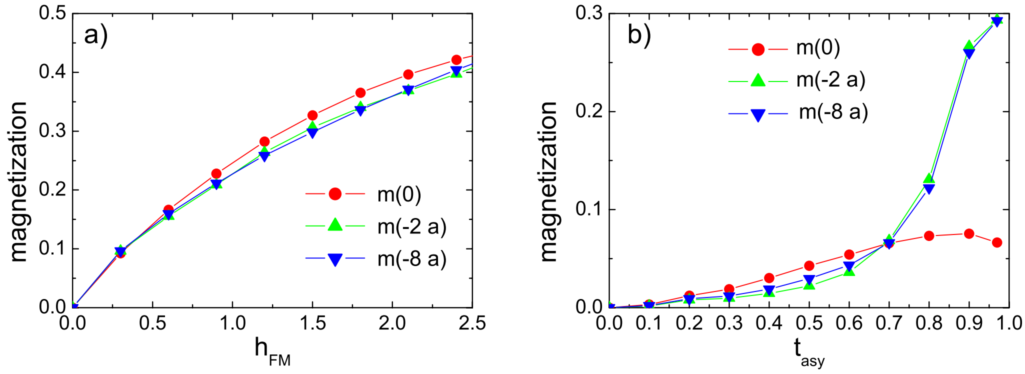

Figure 4. Evolution of the magnetization at different positions with respect to the interface as a function of the exchange field h_{FM} (panel (a)) and the asymmetric ratio $t_{asy} = 1 - t_{FM\downarrow}/t_{FM\uparrow}$ (panel (b)).

Here we have introduced the on-site pairing amplitude $\Delta_{\mathbf{i}} = \langle c_{\mathbf{i}\uparrow} c_{\mathbf{i}\downarrow} \rangle$, with the average $\langle K \rangle$ indicating the expectation value of the operator K over the ground state. Hence, $\Delta_{\mathbf{i}}$ and the on-site magnetization $m_{\mathbf{i}} = \frac{1}{2}(\langle n_{\mathbf{i}\uparrow} \rangle - \langle n_{\mathbf{i}\downarrow} \rangle)$ are the order parameters (OP) to be determined self-consistently.

By means of the Hartree–Fock decoupling it is possible to rewrite the Hamiltonian in a quadratic form and then, within the usual scheme based on the Bogoliubov–de Gennes transformation, to get the self-consistent solution for the magnetic and the superconducting order parameters. Such an approach has been performed for a system of size $L_x \times L_y$ with $L_x = L_y = 120$. We have also performed simulations for larger sizes of the junction without getting any significant change in the results. This means that for $L_x = 120$ we are already in a limit where the length of the system is larger than the characteristic scales associated with the magnetic and the superconducting coherence lengths. Moreover, for convenience the site $x = 0$ has been selected as the position of the interface so that a negative (positive) value for the x coordinate indicates a site belonging to the FM (SC) side of the junction, respectively. All the distances are expressed in units of the interatomic length a. Hereafter, the site dependence will be explicit only for the x coordinate, as along the y direction the system is translationally invariant and then uniform.

For analysing the proximity effect, the following strategy is adopted: (i) the microscopic parameters are chosen in the superconducting side in a way to have a non-zero solution for the s-wave order parameter, (ii) the value of U_{FM} is below the Stoner threshold, (iii) both the field h_{FM} and the ratio $t_{FM\downarrow}/t_{FM\uparrow}$ are varied to span all the possible conditions for the magnetization state of the FM side and (iv) the local density for the majority and the minority spin components as well as the local pairing amplitude are determined self-consistently at all the distances from the interface.

For the ferromagnet with spin-dependent bandwidth, it is convenient to introduce a scaled parameter for the asymmetry ratio defined as $t_{asy} = 1 - t_{FM\downarrow}/t_{FM\uparrow}$. Such a term varies from zero to one when the magnetization goes from zero to the maximum allowed value. To explicitly analyse the dependence of the order parameters, we have chosen the following values for the chemical potential, the coupling terms and the charge transfer amplitude at the interface: $\mu = 0.35$, $U_{SC} = -2.0$, $U_{FM} = 1.2$, $t_T = 1.0$. All the energies are expressed in units of $t_{SC} = t_{FM\uparrow} = 1$.

We start the analysis of the results by observing that the two types of ferromagnet FM1 and FM2 considered here exhibit a different behaviour of the magnetization as a function of the relative control parameters. In figure 4 we have reported the evolution of the magnetization at the interface and close to it for the two types of ferromagnet. As one can notice, the qualitative behaviour is different for the two cases. For the exchange field ferromagnet, the magnetization has a monotonic trend that is not much influenced by the distance from the interface. Still, the magnetization slope is quickly growing at small exchange field with $a \sim \sqrt{h_{FM}}$ dependence. For the spin-dependent bandwidth ferromagnet, the magnetization has a different behaviour when comparing the value at the interface with those away from it. Most importantly, while at small values of the asymmetric ratio the magnetization is slowly growing, when $t_{asy} \geqslant 0.7$ it exhibits a significant enhancement, especially when it moves away from the interface. Furthermore, even in the limit of zero bandwidth for the minority spin component, the magnetization does not reach the saturation value. This behaviour is due to the fact that, by reducing the bandwidth of the minority spin electrons, one does not change at all the total spin density for the majority component. The fast increase is related to the structure of the density of states for the majority spin electrons.

Similar features also occur by inspecting the density of the minority spin electrons as a function of the distance from the interface for the two types of itinerant ferromagnet (see figures 5(a) and (b)). Comparing the case of the exchange field with that of the asymmetric spin-dependent bandwidth, one can notice that there is a slight not uniform spatial modulation that probably reflects the corresponding variation of the superconducting order parameter. Moreover, while there is a smooth change in the value of the minority spin density as the field h_{FM} increases, the evolution as a function of t_{asy} reflects the quick change of the magnetization as one can see approaching the limit $t_{asy} = 1$.

Let us consider now the changes in the superconducting pairing amplitude as related to the above-mentioned characteristics of the two types of ferromagnet considered. In the conventional view of the proximity effect between a

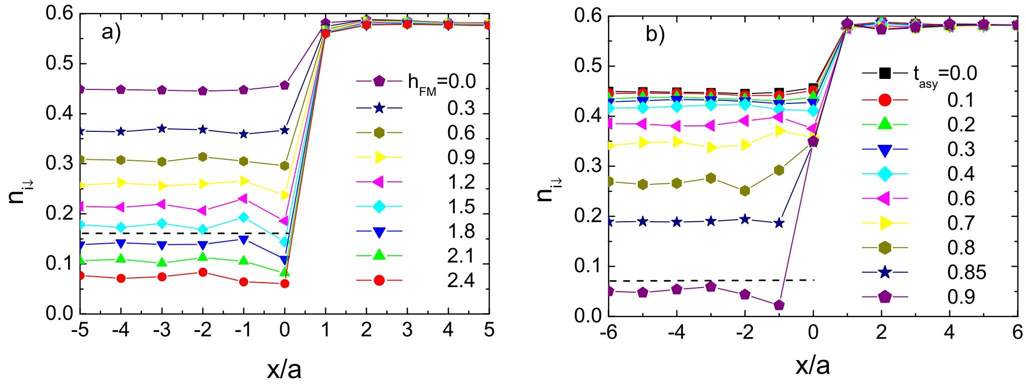

Figure 5. Spatial evolution of the minority spin component near the interface as a function of the ferromagnetic control parameters h_{FM} (panel (a)) and $t_{asy} = 1 - t_{FM\downarrow}/t_{FM\uparrow}$ (panel (b)). The dashed lines indicate the crossover value of the minority spin density below which the proximity behaviour gets exponentially damped within a distance of a few atomic sites from the interface.

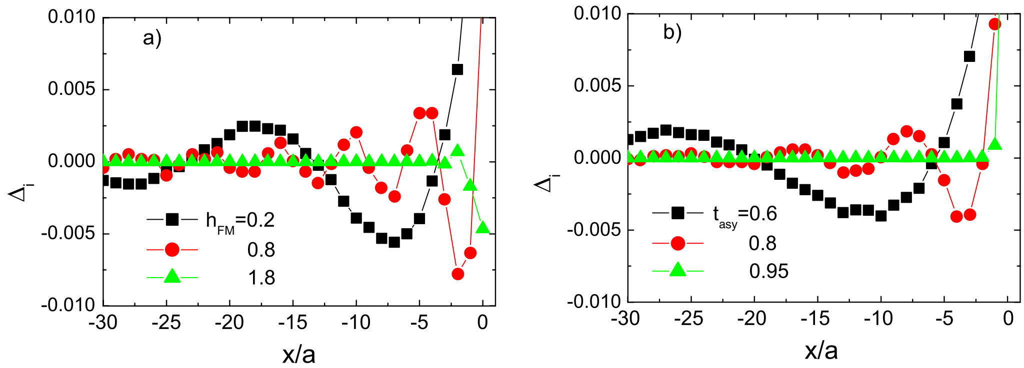

Figure 6. Spatial evolution of the superconducting pairing amplitude within the ferromagnetic side of the junction at different values of the ferromagnetic control parameters h_{FM} (panel (a)) and $t_{asy} = 1 - t_{FM\uparrow}/t_{FM\downarrow}$ (panel (b)).

superconductor and a ferromagnet, the leaking of Cooper pairs into the ferromagnet is marked by an oscillating behaviour of the order parameter whose length is inversely proportional to the exchange field. Here, from the investigation of the spatial evolution of the superconducting pairing amplitude one can observe that in the regime of weak ferromagnetism the pairing amplitude exhibits oscillations with a period that decreases as the value of the control parameters (i.e. h_{FM}, U_{FM}, t_{asy}) increases (see figure 6). This interesting result indicates a kind of universal behaviour for the proximity effect that turns out to be independent from the mechanisms behind the generation of the ferromagnetic order itself.

Nevertheless, a closer inspection reveals that differences emerge when the strength of the ferromagnetism increases so that both the magnetization grows and the minority spin density tends to zero. Strictly speaking, when one of the two components that form the singlet Cooper pair is vanishing, the probability for this pair to penetrate the ferromagnetic side becomes zero. Due to such a simple argument, we do expect that the proximity effect breaks down when approaching the limit of vanishing density for the minority spin component. Such a breakdown is marked by a crossover from an oscillating behaviour of the pairing amplitude to an exponentially damped profile. Following the evolution of the pairing amplitude as a function of the exchange field, one can observe that the breakdown of the proximity effect occurs when the minority spin density is about 0.15, which is away from the limit where $\langle n_{\mathbf{i}\downarrow} \rangle \sim 0$. On the other hand, for the case of an asymmetric spin-dependent bandwidth ferromagnet, the crossover from oscillating to exponentially damped behaviour occurs only when the asymmetric ratio t_{asy} overcomes a critical value that is about 0.9 with a correspondent amplitude of $\langle n_{\mathbf{i}\downarrow} \rangle \sim 0.05$.

4. Conclusions

In conclusion, we have considered the competition between spin singlet pairing and itinerant ferromagnetism whose magnetization is yielded by a relative shift of the bands with opposite spin polarization or by an asymmetric spin-dependent bandwidth. We used the exact solution for an effective reduced BCS model system where spin singlet pairing in the Cooper channel and ferromagnetic correlations form near the Fermi level. The analysis of the exact solution provides a view of the general structure for the quantum eigenstate for a ferromagnetic superconductor. Such topological features, with a sharp separation between the spin polarized and the paired sector in the energy space, turn out to be independent of the mechanism that is behind the generation of the ferromagnetic order. Then, we have investigated the role played by the different types of ferromagnet for the proximity effect within a bilayer junction made of a ferromagnet interfaced with a superconductor. The analysis has been explicitly carried out for the clean limit case in the regime of large transparency at the interface. The main qualitative outcome is that the behaviour of the pairing amplitude within the ferromagnetic side is qualitatively similar for both types of ferromagnets considered in the regime of weak ferromagnetism. Quantitative changes occur when the strength of the ferromagnetism increases. The leaking of Cooper pairs within the ferromagnet turns out to be strongly suppressed for the case of the spin exchange field away from the limit of vanishing minority spin electron component. This is not the case for a ferromagnet with spin-dependent bandwidth where the suppression of the proximity effect is less severe.

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